

1. Give the standard definitions of the following notions:
 - a. Logical consequence (邏輯結果) (5 points):
 - b. Logical equivalence (邏輯等值) (5 points)
 - c. Truth-functional connectives (真值函數連接詞) (5 points):
 - d. Closed sentence (封閉語句) (5 points)
2. Decide the truth values of the following statements. (2 points each)
 - (1) If an argument is invalid, then at least one premise of the argument must be false. (如果一個論證是無效的，則該論證至少必須有一個前提為假。)
 - (2) All sound arguments are valid. (所有健全的論證都是有效的)
 - (3) If all the premises and the conclusion of an argument are false, then the argument must be invalid. (如果一個論證的所有前提和結論都為假，則該論證必然是無效的。)
 - (4) No true sentence can logically imply any false sentence. (沒有一個真的語句能夠邏輯上蘊含任何的假語句。)
 - (5) A sound argument must have true conclusion. (健全的論證一定有真的結論。)
 - (6) When the conclusion of an argument is a tautology, that argument need not be valid. (當論證的結論是套套邏輯時，該論證不必然是有效的。)
 - (7) When an argument has a self-contradictory premise or inconsistent premises, it is valid. (當論證前提不一致或包含矛盾前提時，該論證是有效的。)
 - (8) Every tautology is logically implied by every sentence. (每個句子都邏輯上蘊含套套邏輯。)
 - (9) Every argument whose conclusion is identical with one of its premises is valid. (結論與某一前提相同的論證是有效的。)
 - (10) There are 16 different truth functions for two arguments. (一共有十六種不同的二位真值函數。)
3. Translate the following sentence into symbolic form: (5 points for each)
 - (1) You are not permitted to enter if you are not accompanied by an adult. (如果沒有成人作伴，是不得入場的。)(A: You are accompanied by an adult; P: You are permitted to enter.)
 - (2) It's true that either all logic students are logical or they are all unpopular. (這是真的：或者邏輯課學生都很邏輯，或者他們都很不受歡迎。)(Let $Sx = "x \text{ is a logic student}"; Lx = "x \text{ is logical}";$ and $Px = "x \text{ is popular}";$)
 - (3) Everything that is loved by John is loved at least by someone. (每一個約翰喜愛的東西都至少被某些人所喜愛。)(Let $Lxy = "x \text{ loves } y"; j = "John"; Mx = "x \text{ is a human being}";$)

(4) Everyone loves someone, but not vice versa. (每個人都愛至少一個人，但反之不然) (Let $Lxy = "x \text{ loves } y"$; $Mx = "x \text{ is a human being}"$.)

4. Prove that the following arguments are valid, using whatever method you know. (10 points each.)

- a. 1. $(y)\neg By \supset \neg(\exists x)Ax$
 2. $(\exists x)[Ax \ \& \ (y)(By \supset Cax)] \ \therefore (\exists x)(\exists y)Cxy$
- b. 1. $(x)(\neg Cx \vee \neg Bx)$
 2. $(\exists z)(Az \ \& \ Cz)$
 3. $(y)[\neg(Ay \supset Dy) \supset By]$ $\therefore (\exists z)Dz$

5. Prove that the following sentences are theorems. (10 points each.)

- a. $((\exists z)Fz \supset Ga) \supset (x)(Fx \supset Ga)$
 b. $(x)(\exists y)Rxy = (w)(\exists u)Rwu$