

(共 12 題，總計 100 分) (18 條規則附錄於第二頁)

符號說明：• : and \vee : or \sim : not \supset : If... then... \equiv : ...if and only if...

(也可用相當的符號，如 \wedge , \vee , \neg , \rightarrow , \leftrightarrow)

(關於(II)大題，考生可以用類似的證明系統，但是使用 truth tree 或 semantic tableau 或 analytic tableau 等方法者，除非另行證明該方法等價於這裡使用的系統，將酌予扣分。)

(I) 請將下列語句符號化。(合計 30 分)

(1) John does not love anyone who is not afraid of Henry. (6 分)

(John: j; Henry: h; Axy: "x is afraid of y"; Lxy: "x loves y"; domain: all humans)

(2) No one fears death. (6 分)

(Hx="x is a human"; Fx="x fears death"; domain: unrestricted)

(3) It is not the case that some Martians don't eat cows. (6 分)

(Mx="x is a Martian"; Ex="x eats cows"; domain: unrestricted)

(4) If someone is too noisy, then everyone in the room will dislike that person. (6 分).

(Rx="x is in the room"; Nx="x is too noisy"; Dxy="x dislikes y"; domain: all humans)

(5) None but those who study hard will pass the final. (6 分) (Dx= "x studies hard";

Bx= "x pass the final; domain: all students)

(II)使用 18 條規則或 CP (Conditional Proof) 、 IP(Indirect Proof) 、 QN (Quantifier Negation) 、 UI(Universal Instantiation) 、 EI(Existential Instantiation) 、 UG(Universal Generalization) 、 EG(Existential Generalization)證明下列論證。(合計 45 分)

(6) 1. $(x) Fx \vee (x) \sim Gx$

2. $\sim(x) Fx$

3. $(x) (Dx \supset Gx) / \therefore (\exists x) \sim Dx$ (9 分)

(7) 1. $(\exists x)(y)Fxy$

2. $(y)(\exists x)(Fyx \supset Gxy) / \therefore (\exists y)(\exists x)Gyx$ (9 分)

(8) 1. $(\exists x)[(Fx \vee Gx) \supset Kx]$

2. $(x)[(Hx \vee Kx) \supset Lx] / \therefore (\exists x)(Gx \supset Lx)$ (9 分)

(9) 1. $\sim(x)(Dx \vee Bx)$

2. $(\exists x)\sim Dx \supset (y)(Ay \supset By) / \therefore \sim(x)Ax$ (9 分)

(10) 1. $(\exists x) Fx$

2. $(x) \sim Fx \quad / \therefore \{(\exists x) Gx \equiv (x) [(Lx \equiv Gx) \equiv Hx]\}$ (9 分)

(III) 語意(semantics)與證明系統的健全性(soundness) (25 分)

(11)請證明 $B \supset (B \supset A)$ 不是古典命題邏輯(例如使用 18 條規則及 CP, IP 的證明系統)下的定理(theorem) (5 分), 並說明為何下列“證明”有誤 (7 分)。

1. A AP
2. $A \vee \sim B$ 1 Add
3. B AP
4. $\sim B \vee A$ 2 Comm
5. $\sim \sim A$ 1 DN
6. $B \supset A$ 4 Impl
7. $A \supset (B \supset A)$ 1-6 CP
8. A 5 DN
9. $B \supset A$ 7,8 MP
10. $B \supset (B \supset A)$ 3-9 CP

(12) 舉反例說明下列論證無效(invalid): $(x)(\exists y)Lxy / \therefore (\exists y)(x)Lxy$ (6 分), 並說明為何下列“證明”有誤 (7 分)。

1. $(x)(\exists y)Lxy / \therefore (\exists y)(x)Lxy$
2. $(\exists y)Lxy$ 1. UI
3. Lxy 2. EI
4. $(x)Lxy$ 3. UG
5. $(\exists y)(x)Lxy$ 4. EG

附錄：18 條規則 (Appendix: the 18 valid argument forms)

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|---|---|--|-------------------------|
| 1. MP $p \supset q$ | 2. MT $p \supset q$ | 3. DS $p \vee q$ | $p \vee q$ |
| $p / \therefore q$ | $\sim q / \therefore \sim p$ | $\sim p / \therefore q$ | $\sim q / \therefore p$ |
| 4. Simp $p \bullet q / \therefore p$ | 5. Conj p | 6. HS $p \supset q$ | |
| $p \bullet q / \therefore q$ | $q / \therefore p \bullet q$ | $q \supset r / \therefore p \supset r$ | |
| 7. Add $p / \therefore p \vee q$ | 8. CD $p \supset q$ | | |
| | $r \supset s$ | | |
| | $p \vee r / \therefore q \vee s$ | | |
| 9. DN $p :: \sim \sim p$ | 10. DeM $\sim (p \bullet q) :: \sim p \vee \sim q$ | 11. Comm $(p \vee q) :: (q \vee p)$ | |
| | $\sim (p \vee q) :: \sim p \bullet \sim q$ | $(p \bullet q) :: (q \bullet p)$ | |
| 12. Assoc $[p \vee (q \vee r)] :: [(p \vee q) \vee r]$ | 13. Dist $[p \bullet (q \vee r)] :: [(p \bullet q) \vee (p \bullet r)]$ | | |
| $[p \bullet (q \bullet r)] :: [(p \bullet q) \bullet r]$ | $[p \vee (q \bullet r)] :: [(p \vee q) \bullet (p \vee r)]$ | | |
| 14. Contra $(p \supset q) :: (\sim q \supset \sim p)$ | 15. Impl $(p \supset q) :: \sim p \vee q$ | | |
| 16. Exp $[(p \bullet q) \supset r] :: [p \supset (q \supset r)]$ | 17. Taut $p :: (p \bullet p)$ | | |
| | $p :: (p \vee p)$ | | |
| 18. Equiv $(p \equiv q) :: [(p \supset q) \bullet (q \supset p)]$ | | | |
| $(p \equiv q) :: [(p \bullet q) \vee (\sim p \bullet \sim q)]$ | | | |