初階邏輯試題

In this test,
"$\neg$" means "not",
"$\wedge$" means "and",
"$\vee$" means "or",
"$\rightarrow$" means "if...then...",
"$\leftrightarrow$" means "if and only if",
"$\forall x$" means "for all x" and
"$\exists x$" means "for some x".

I. For each of the following statements, just write **True** or **False**. No explanation is needed. **(30 points; 3 points each)**

1. If neither P nor Q is a contradiction, then $P \wedge Q$ is not a contradiction.
2. Let R be a binary predicate. Then R(a, b) logically implies R(b, a).
3. P logically implies Q if and only if P logically implies $P \rightarrow Q$.
4. If A and B are consistent and A and C are consistent, then B and C must be consistent.
5. $A \vee (A \wedge [(B \wedge C) \rightarrow (D \vee E)])$ is equivalent to $A \wedge (A \vee F)$.
6. $(P \leftrightarrow Q) \vee (P \leftrightarrow R) \vee (R \leftrightarrow Q)$ is not a tautology.
7. If we add a new premise to an invalid argument, the resulting argument will still be invalid.
8. $A \leftrightarrow (A \leftrightarrow A)$ is a tautology.
9. If we add a new premise to a valid argument, the resulting argument may be invalid.
10. If P does not imply Q, then P must imply $\neg Q$.

II. Let "Lxy" stand for "x loves y",
"Hxy" stand for "x hates y",
"Px" stand for "x is a philosopher" and
"M" stand for "Mary".

Please symbolize the following two sentences. **(20 points; 10 points each)**

(1) There is a philosopher who hates another philosopher and who is loved by exactly two philosophers.
(2) Mary hates every philosopher unless at least three philosophers love her.

III. Please give counterexamples to the following two **invalid** arguments. **(20 points; 10 points each)**

(1) $\forall x(Px \lor Rx)$ / $\therefore$ $\forall xPx \lor \forall xRx$

(2) $\exists x(Px \to \exists yRy)$ / $\therefore$ $\exists xPx \to \exists yRy$

IV. Please prove the following two **valid** arguments. (You may use the system on the next page. But virtually all formal proof systems are acceptable; just make your proof as clear as possible). **(30 points; 15 points each)**

(1) / $\therefore$ $\exists x \forall y Pxy \to \forall y \exists x Pxy$

(2) $\exists x(Px \lor Qx) \to \forall xRx$ / $\therefore$ $\forall x \exists y((Px \to Rx) \land (Qy \to Rx))$