

In this test,

“ \neg ” means “not”,

“ \wedge ” means “and”,

“ \vee ” means “or”,

“ \rightarrow ” means “if...then...”,

“ \leftrightarrow ” means “if and only if”,

“ $\forall x$ ” means “for all x” and

“ $\exists x$ ” means “for some x”.

I. True or False

Please answer each question by writing **True** or **False**. No explanation is needed. (15 points; 3 points each)

1. Two different predicates must have different interpretations (extensions).
2. $P(a,b,a) \leftrightarrow P(b,a,b)$ logically implies $a = b$.
3. If $\exists x \exists y (P(x) \wedge P(y))$ is true in a model, then the domain of that model must contain at least two members.
4. $\forall x (P(x) \rightarrow Q(x))$ is logically equivalent to $\forall x P(x) \rightarrow \forall x Q(x)$.
5. $[(A \wedge B) \rightarrow C] \rightarrow [B \rightarrow (A \rightarrow (D \rightarrow C))]$ is a tautology.

II. Symbolization

Let “ Lxy ” stand for “x loves y”,

“ Hxy ” stand for “x hates y” and

“ Px ” stand for “x is a philosopher”.

Please symbolize the following two sentences. (30 points; 15 points each)

- (1) There are exactly two philosophers who love the same philosopher.
- (2) Every philosopher hates every other philosopher unless someone who is not a philosopher loves them.

III. Please give counterexamples to the following two **invalid** arguments. (30 points; 15 points each)

(1) $\forall x (Px \rightarrow Rx) \therefore \exists x Rx$

(2) $\exists x (Px \rightarrow \forall y Ry) \therefore \exists x Px \rightarrow \forall y Ry$

IV. Please prove the following **valid** argument. (You may use the system on the next page. But virtually all formal proof systems are acceptable; just make your proof as clear as possible). **(25 points)**

$$\forall x(\neg((Px \leftrightarrow Qx) \leftrightarrow Px) \leftrightarrow \neg(Rx \leftrightarrow Qx)), \exists x(R(x) \rightarrow Q(x)) / \therefore \exists x Qx$$