國立中正大學 碩士班甄試 邏輯試題 **2008/11/21**

In this test,
"$\neg$" means "not",
"$\wedge$" means "and",
"$\vee$" means "or",
"$\rightarrow$" means "if...then...",
"$\leftrightarrow$" means "if and only if",
"$\forall x$" means "for all x" and
"$\exists x$" means "for some x".

I. True or False

Please answer each question by writing **True** or **False.** No explanation is needed. **(15 points; 3 points each)**

1. Two different predicates must have different interpretations (extensions).
2. $P(a,b,a) \leftrightarrow P(b,a,b)$ logically implies $a = b$.
3. If $\exists x\exists y(P(x)\wedge P(y))$ is true in a model, then the domain of that model must contain at least two members.
4. $\forall x(P(x)\rightarrow Q(x))$ is logically equivalent to $\forall xP(x)\rightarrow\forall xQ(x)$.
5. $[(A\wedge B)\rightarrow C]\rightarrow[B\rightarrow(A\rightarrow(D\rightarrow C))]$ is a tautology.

II. Symbolization

Let "Lxy" stand for "x loves y",
"Hxy" stand for "x hates y" and
"Px" stand for "x is a philosopher".

Please symbolize the following two sentences. **(30 points; 15 points each)**

(1) There are exactly two philosophers who love the same philosopher.
(2) Every philosopher hates every other philosopher unless someone who is not a philosopher loves them.

III. Please give counterexamples to the following two **invalid** arguments. **(30 points; 15 points each)**

(1) $\forall x(Px\rightarrow Rx)$ / $\therefore\exists xRx$
(2) $\exists x(Px\rightarrow\forall yRy)$ / $\therefore\exists xPx\rightarrow\forall yRy$

IV. Please prove the following **valid** argument. (You may use the system on the next page. But virtually all formal proof systems are acceptable; just make your proof as clear as possible). **(25 points)**

$\forall x(\neg((Px \leftrightarrow Qx) \leftrightarrow Px) \leftrightarrow \neg(Rx \leftrightarrow Qx)), \exists x(R(x) \rightarrow Q(x)) \; / \therefore \exists x Qx$