In this test,
"¬" means "not",
"∧" means "and",
"∨" means "or",
"→" means "if... then...",
"↔" means "if and only if",
"∀x" means "for all x" and
"∃x" means "for some x".

I. True or False

Please answer each of the following questions by writing down T or F. No explanation is needed. **(30 points; 3 points each)**

1. If we add a new premise to a valid argument, the resultant argument may be invalid.
2. If c and c' are different constants, then the interpretation of c and the interpretation of c' cannot be the same.
3. ∀x (Ax ∨ Bx) is logically equivalent to ∀x Ax ∨ ∀x Bx.
4. If A logically implies B, then either A is a contradiction or B is a tautology.
5. If ∃x∃y (Px ∧Ry ) is true in a model, then there are at least two different objects in the domain of that model.
6. If P is not logically equivalent to Q, then P is logically equivalent to ¬Q.
7. If A is inconsistent with B while B is consistent with C, then A is inconsistent with C.
8. If A → B is a contradiction, then A is a tautology.
9. A ∨ (¬((B ∧ ¬D)→ (C ∧ (E ∨ C))) → (D ↔ A)) is a tautology.
10. The interpretation of a predicate cannot be an empty set.

II. Let "Lxy" stand for "x loves y",
"Hxy" stand for "x hates y",
"Px" stand for "x is a philosopher" and
"m" stand for "Mary".

Please symbolize the following two sentences. **(20 points; 10 points each)**

1. Marry hates someone who is not a philosopher unless all philosophers love her.
2. All philosophers hate exactly one non-philosopher who loves Mary.

III. Please give counterexamples to the following two invalid arguments. (20 points; 10 points each)

1. $\forall x\, Px \rightarrow \forall x\, Qx \;/ \therefore \forall x\, (Px \rightarrow Qx)$
2. $\exists x \forall y\, (\neg x{=}y \rightarrow Rxy)$, $\neg\exists x\, Rxx$, $\forall x \forall y \forall z\, ((Rxy \wedge Ryz) \rightarrow Rxz)$, $\forall x \exists y\, Rxy$ $/ \therefore \exists x \forall y\, (\neg x{=}y \rightarrow Ryx)$

IV. Please prove the following three valid arguments. (30 points; 10 points each)

1. $/ \therefore ((P \rightarrow Q) \rightarrow Q) \rightarrow Q$
2. $\forall x \forall y \forall z\, (Rxy \rightarrow (Rxz \rightarrow Ryz))$, $\forall x\, Rxx \;/ \therefore \forall x \forall y \forall z\, (Rxy \rightarrow (Ryz \rightarrow Rxz))$
3. $\forall x\, \neg(Px \leftrightarrow Qx)$, $\exists x\, (Px \vee (Rx \wedge Px)) \;/ \therefore \forall x\, Qx \leftrightarrow (Ra \wedge \neg Ra)$