

In this test,

" \neg " means "not",

" \wedge " means "and",

" \vee " means "or",

" \rightarrow " means "if... then...",

" \leftrightarrow " means "if and only if",

" $\forall x$ " means "for all x" and

" $\exists x$ " means "for some x".

I. True or False

Please answer each of the following questions by writing down T or F. No explanation is needed. (30 points; 3 points each)

1. If we add a new premise to a valid argument, the resultant argument may be invalid.
2. If c and c' are different constants, then the interpretation of c and the interpretation of c' cannot be the same.
3. $\forall x (Ax \vee Bx)$ is logically equivalent to $\forall x Ax \vee \forall x Bx$.
4. If A logically implies B , then either A is a contradiction or B is a tautology.
5. If $\exists x \exists y (Px \wedge Ry)$ is true in a model, then there are at least two different objects in the domain of that model.
6. If P is not logically equivalent to Q , then P is logically equivalent to $\neg Q$.
7. If A is inconsistent with B while B is consistent with C , then A is inconsistent with C .
8. If $A \rightarrow B$ is a contradiction, then A is a tautology.
9. $A \vee (\neg((B \wedge \neg D) \rightarrow (C \wedge (E \vee C))) \rightarrow (D \leftrightarrow A))$ is a tautology.
10. The interpretation of a predicate cannot be an empty set.

- II. Let " Lxy " stand for "x loves y",
" Hxy " stand for "x hates y",
" Px " stand for "x is a philosopher" and
" m " stand for "Mary".

Please symbolize the following two sentences. (20 points; 10 points each)

1. Marry hates someone who is not a philosopher unless all philosophers love her.
2. All philosophers hate exactly one non-philosopher who loves Mary.

III. Please give counterexamples to the following two invalid arguments. (20 points; 10 points each)

1. $\forall x Px \rightarrow \forall x Qx / \therefore \forall x (Px \rightarrow Qx)$
2. $\exists x \forall y (\neg x=y \rightarrow Rxy), \neg \exists x Rxx, \forall x \forall y \forall z ((Rxy \wedge Ryz) \rightarrow Rxz), \forall x \exists y Rxy$
 $/ \therefore \exists x \forall y (\neg x=y \rightarrow Ryx)$

IV. Please prove the following three valid arguments. (30 points; 10 points each)

1. $/ \therefore ((P \rightarrow Q) \rightarrow Q) \rightarrow Q$
2. $\forall x \forall y \forall z (Rxy \rightarrow (Rxz \rightarrow Ryz)), \forall x Rxx / \therefore \forall x \forall y \forall z (Rxy \rightarrow (Ryz \rightarrow Rxz))$
3. $\forall x \neg(Px \leftrightarrow Qx), \exists x (Px \vee (Rx \wedge Px)) / \therefore \forall x Qx \leftrightarrow (Ra \wedge \neg Ra)$