國立中正大學 碩士班甄試邏輯試題 2012/11/16

In this test,
"¬" means "not",
"∧" means "and",
"∨" means "or",
"→" means "if...then...",
"↔" means "if and only if",
"∀x" means "for all x", and
"∃x" means "for some x".

I. True or False. Please answer each of the following questions simply by writing True or False. (40 points; 4 points each)

1. If the conclusion of an argument is a contradiction, then that argument is invalid.
2. If $\exists x\ \exists y\ (Px \wedge Py)$ is true in a model, then the domain of that model must contain at least two members.
3. $\forall x\ (Px \vee Qx)$ is logically equivalent to $\forall x\ Px \vee \forall x\ Qx$.
4. Two different constants must have different interpretations.
5. P and Q are inconsistent if and only if P and Q have different truth values.
6. $A \leftrightarrow \neg ((B \leftrightarrow (\neg (A \leftrightarrow C) \leftrightarrow C)) \leftrightarrow B)$ is a tautology.
7. Predicate symbols are logical symbols.
8. If an argument is valid, then it is sound.
9. Suppose A is contingent. If A and B are inconsistent and A and C are inconsistent, then B and C must be inconsistent.
10. If $A \to (B \to (C \to D))$ is not a tautology, then D is not a tautology.

II. Let "Lxy" stand for "x loves y",
"Hxy" stand for "x hates y" and
"Px" stand for "x is a philosopher".
Please symbolize the following sentence. (20 points)

1. All philosophers hate at most one person who is not a philosopher and who loves exactly one philosopher.

III. Please prove the following two valid arguments (Any proof system is acceptable; just try to make your proofs as clear as possible). (40 points; 20 points each)

1. $(\neg (A \leftrightarrow B) \vee (A \wedge B)) \to (C \vee (A \wedge C))$ /∴ $(A \vee B) \to C$
2. $\neg (\forall x\ \exists y\ (\neg Sxy \leftrightarrow Sxx) \vee \exists y\ \forall x\ Pxy) \vee (\neg\forall x\ \exists y\ (Sxy \leftrightarrow \neg Sxx) \wedge \forall x\ \exists y\ Pxy)$ /∴ $\exists x\ (\exists y\ Sxy \to \forall y\ Sxy)$