中正大學 103 學年度碩士甄試
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- In the following questions, we will use (1) $\sim$ as "negatio, (2) $\wedge$ as "conjunction", (3)$\vee$ as "or", (4) $\rightarrow$ as "implication", and (5)$\leftrightarrow$ as "equivalence", (6) (x) as "for all x", (7) $(\exists x)$ as "for some x", and $\therefore$ for "therefore" in an argument.
- If you want to answer the questions with the symbols you are familiar with, explicitly state the intended meaning of the symbols you are using.
- When proving, you can use the proof system that you are familiar with, but specify the source of your proof system (from which book and who is the author).

1. Translate the following English sentences into well-formed formulas, with English alphabets standing for *atomic sentences* - that is, those sentences that are not built up out of other sentences.(10pts)
   (a) Either Sam will come to the party and Max will not, or Sam will not come to the party and Max will enjoy himself.
   (b) Fiorello goes to the movies only if a comedy is playing.

2. Use truth table method to show the validity of the following argument. (10pt)
   1. $(A \wedge B) \vee C$ 2. $\sim(A \vee B)$ $\therefore C$.

3. Prove that the following argument is a valid argument (no semantic method).(10pt)
   1. $\sim(P \wedge \sim Q)$ 2. $\sim Q \vee M$ 3. $R \rightarrow \sim M$ $\therefore P \rightarrow \sim(R \vee \sim M)$.

4. Prove that $(\sim A \wedge (A \vee B)) \rightarrow B)$ is a tautology (no semantic method).(5pt)

5. Translate the following sentences into well-formed formulas in first-order logic. (15pts)
   (a) Anyone who is persistent can learn logic
   (b) John hates everyone who does not hate himself.
   (c) Everyone loves somebody and no one loves everybody, or somebody loves everybody and someone loves nobody.

6. Prove the validity of the following arguments. (20pt)
   (a) 1. $(x)[(Fx \vee Gx) \rightarrow Hx]$
   2. $(x)[(Hx \vee Kx) \rightarrow Lx] / \therefore (x)(Fx \rightarrow Lx)$

   (b) 1. $(x)(\exists y)Fxy \rightarrow (x)(\exists y)Gxy$
   2. $(\exists x)(y)\sim Gxy / \therefore (\exists x)(y)\sim Fxy$

7. Show that the following arguments are invalid (20pt)
   (a) 1. $(x)(Px \supset Qx)$
       2. $(x)(Qx \supset Rx) / \therefore (x)(Px \wedge Rx)$
   (b) 1. $(x)(\exists y)(Fxy \to Gxy)$
       2. $(x)(\exists y)(Gxy \to Hxy) / \therefore (x)(\exists y)(Fxy \to Hxy)$

8. (10pt)
   Let $\Gamma_0, \Gamma_1, \ldots, \Gamma_n, \ldots$ be a sequence of sets such that $\Gamma_i \subseteq \Gamma_{i+1}$ and $\Gamma^* = \bigcup_{i=0}^{\infty} \Gamma_i$. Prove that if Γ' is a finite subset of Γ^*, then $\Gamma' \subseteq \Gamma_i$ for some i.