中正大學 104 學年度碩士甄試
哲學系 -邏輯

- In the following questions, we will use (1) $\sim$ as "negatio, (2) $\wedge$ as "conjunction", (3)$\vee$ as "or", (4) $\rightarrow$ as "implication", and (5)$\leftrightarrow$ as "equivalence", (6) (x) as "for all x", (7) $(\exists x)$ as "for some x", and $\therefore$ for "therefore" in an argument.
- If you want to answer the questions with the symbols you are familiar with, explicitly state the intended meaning of the symbols you are using.
- When proving, you can use the proof system that you are familiar with, but specify the source of your proof system (from which book and who is the author).

1. Translate the following English sentences into well-formed formulas. (A = John does well in logic, B = John does well in philosophy, C = John does well in all classes, D = John does graduate, E = John does pay tuition.) (10pts)
   (a) John does well in all his classes if he does well in logic and philosophy.
   (b) John does well in all his classes, but he doesn't graduate, if he doesn't pay tuition.

2. Use truth table method to show the validity of the following argument. (10pt)
   1. $P \rightarrow \sim Q$ 2. $Q \vee R$ 3. $P \vee R$ $\therefore R$.

3. Prove that the following argument is a valid argument (no semantic method).(10pt)
   1. $S \vee (T \rightarrow R)$ 2. $S \rightarrow T$ 3. $\sim(T \rightarrow R)$ $\therefore T$.

4. Prove that $(P \wedge (\sim P \vee Q)) \rightarrow Q)$ is a tautology (no semantic method).(5pt)

5. Translate the following sentences into well-formed formulas in first-order logic. (15pts)
   (a) There are physicians who are competent but lack sensitivity.
   (b) Neither all of the Republicans, nor all of the Democrats, supported the President's Health Care plan.
   (c) Every student takes some of the courses that John takes.

6. Prove the validity of the following arguments. (20pt)
   (a) 1. $(\exists x)Fx \vee (\exists x)Gx$
   2. $(x)\sim Fx / \therefore (\exists x)Gx$

   (b) 1. $(\exists x)(y)(Fxy \rightarrow Gyx)$
   $\therefore (x)(\exists y)(Fyx \rightarrow Gxy)$

7. Show that the following arguments are invalid (10pt)
   1. $(x)(Px \rightarrow Qx)$
   2. $(\exists x)\sim Px / \therefore (\exists x)\sim Qx$

8. Let A, B, C be sets, and $A \subset B$ and $B \subset C$, i.e. A is a subset of B, and B is a subset of C. Prove that $A \subset C$, i.e. A is a subset of C. (10pt)

9. Let R be a set of ordered pairs (a, b) for a, b in a set of A, i.e., $R \subset A \times A$, and R^{-1} be the set of ordered pairs (a, b) such that (b, a) are in R. Suppose R is symmetric, i.e., if (a, b) is in R, so is (b, a). Prove that R^{-1} is symmetric too. (10pt)

Answer Key for Exam A

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