# 國立中正大學　碩士班甄試　邏輯試題 2014/11/14

In this test,
"$\neg$" means "not",
"$\wedge$" means "and",
"$\vee$" means "or",
"$\rightarrow$" means "if... then...",
"$\leftrightarrow$" means "if and only if",
"$\forall x$" means "for all $x$" and
"$\exists x$" means "for some $x$".

## I. True or False

Please answer each question by writing True or False. No explanation is needed. (30 points; 3 points each)

1. $(A \leftrightarrow B) \wedge (B \leftrightarrow C)$ is logically equivalent to $A \leftrightarrow (B \leftrightarrow C)$
2. If $A \wedge B$ is a contradiction, $B \wedge C$ is a contradiction and $B$ is not a contradiction, then $A$ is logically equivalent to $C$.
3. If $A$ implies $B$, then $A$ and $C$ together also imply $B$.
4. $(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$ is a tautology.
5. If $A \rightarrow B$ is a tautology and $A$ is not a contradiction, then $B$ is a tautology.
6. In the formula $\exists x \forall x Px$, the "$x$" occurring in "$Px$" is bounded by "$\exists x$".
7. $\forall x Px$ is logically equivalent to $\forall y Py$.
8. $Px$ is logically equivalent to $Py$.
9. The interpretations of two different constants cannot be the same.
10. $\forall x \exists y Pxy$ is logically equivalent to $\exists y \forall x Pxy$

## II. Symbolization

Let ''$Px$" stand for "$x$ is a philosopher", "$Lxy$" stand for "$x$ loves $y$" and "$Hxy$" stand for "$x$ hates $y$". Please symbolize the following two sentences. (30 points; 15 points for each)

1. All philosophers love someone who is not a philosopher, unless someone who is not a philosopher hates all philosophers.
2. If there are exactly two philosophers loved by all philosophers, then some philosopher hates at most two people who are not philosophers.

## III. Please give a counterexample to the following invalid argument. (20 points)

1. $\forall x(Px \vee Qx) / \therefore \forall x Px \vee \forall x Qx$

## IV. Please prove the following valid argument.

Virtually all formal proof systems are acceptable; just make your proof as clear as possible. (20 points)

1. $\exists x \forall y(Sxy \leftrightarrow Sxx) / \therefore \exists x(\forall y Sxy \vee \forall y \neg Sxy)$