In this test,

"¬" means "not",

"∧" means "and",

"∨" means "or",

"→" means "if...then...",

"↔" means "if and only if",

"∀x" means "for all x", and

"∃x" means "for some x".

I. True or False

Please answer each of the following questions simply by writing **True** or **False. (20 points; 2 points each)**

1. (A↔¬E)↔[¬(A↔F)↔(E↔F)] is a tautology.
2. ∃x(P(x)∨R(x)) is logically equivalent to ∃xP(x)∨∃xR(x).
3. If every woman loves some men, then every woman loves some men who love her. Hence no woman can love some men who do not love her.
4. A valid argument cannot have false premises.
5. Suppose that P is not consistent with S. Then P cannot imply S.
6. Suppose that all philosophers are smart. Then some smart people are philosophers.
7. If formulas A and B are equivalent, then A and B must be the same formula.
8. Suppose that f is a unary function symbol and R is a unary predicate. Then f(x)→R(x) is a grammatical (well-formed) formula.
9. Suppose that I say that I will marry my girlfriend if she gives me one million dollars. Then I am lying if I will still marry her without receiving that money.
10. If P and R are two different unary predicates, then ∃x¬(P(x)↔R(x)) is always the case.

II. Please give counterexamples to the following two **invalid** arguments. **(30 points; 15 points each)**

(1) ∃x((Px∨Qx)→∃yRy) /∴ ∃xPx→∃yRy

(2) ∀x¬R(x, x)∧∀x∃yR(x, y)∧∀x∀y∀z(R(x, y)→(R(y, z)→R(x, z)))
  /∴∀x∃y(R(x, y)∧∀z((z≠y∧R(x, z))→R(y, z)))

III. Please symbolize the following two sentences. **(20 points; 10 points each)**

(1) Any great philosopher admires exactly two persons who are not philosophers.

(2) At most two students will take the course unless it is guaranteed that everyone taking that course won't be failed.

IV. Please prove the following **valid** arguments. (Virtually all formal proof systems are acceptable. Just make your proofs as clear as possible). **(30 points; 15 points each)**

(1) $(D\lor E)\rightarrow[A\land((B\rightarrow C)\lor A)]$, $F\rightarrow\neg(A\leftrightarrow G)$ /∴ $G\rightarrow(D\rightarrow\neg F)$

(2) $\forall x\neg[(P(x)\leftrightarrow R(x))\leftrightarrow Q(x)]$, $\exists x\forall y(\neg R(x)\lor S(x, y))$ /∴
$\forall x(P(x)\leftrightarrow\neg Q(x))\rightarrow\exists x(Q(x)\rightarrow S(x, x))$