In this test, "¬" means "not", "∧" means "and", "∨" means "or", "→" means "if...then...", "↔" means "if and only if", "∀x" means "for all x", and "∃x" means "for some x".

I. Which of the following statements are true? (Answer each question by writing T (true) or F (false). No explanation is needed.) (20 points; 2 points each)

1. If $A \to B$ is a contradiction, then A is not a contradiction.
2. A valid argument can have a false conclusion.
3. $A \wedge B$ and $B \wedge A$ are the same sentential expression.
4. $[\neg C \leftrightarrow ((A \vee B) \leftrightarrow C)] \to [(A \wedge D) \to E]$ is a tautology.
5. If A and B are consistent and A and C are consistent, then B and C must be consistent.
6. If we add a new premise to a valid argument, the resultant argument may be invalid.
7. In predicate logic, $R(a, b)$ logically implies $R(b, a)$.
8. If A and B are unary predicates and R is a binary predicate, then $R(A(x), B(x))$ is a well-formed formula in predicate logic.
9. All sound arguments are valid.
10. If neither A nor B is a contradiction, then $A \wedge B$ is not a contradiction.

II. Please symbolize the following sentences. (20 points; 10 points each)

1. John will not marry Mary unless Joe loves him or her. (Let "A" stand for "John will marry Mary", "B" stand for "Joe loves John" and "C" stand for "Joe loves Marry".)
2. Every child has exactly two toys. (Let "$C(x)$" stand for "x is a child", "$T(x)$" stand for "x is a toy" and "$H(x, y)$" stand for "x has y".)

III. Please give counterexamples to the following invalid arguments. (30 points; 15 points each)

1. $A \to (B \vee C), \neg(B \to A) / \therefore C \to A$
2. $(A \to \neg B) \wedge \neg C, B \to (\neg A \to C) / \therefore (A \leftrightarrow B) \leftrightarrow C$

IV. Please prove the following valid arguments (all formal proof systems are acceptable; just try to make your proofs as clear as possible). (30 points; 15 points each)

1. $A \leftrightarrow B, B \vee C, A \to \neg C / \therefore (C \leftrightarrow B) \leftrightarrow (A \wedge C)$
2. $/ \therefore \exists x((\exists y P(y) \to \exists z R(z)) \to (P(x) \to R(x)))$