國立中正大學 2004/6/13 博士班入學邏輯試題 (12 problems, total 100 points)

符號說明: ~ 表示 not; • 表示 and; ∨ 表示 or (inclusive or); ⊃表示 if… then …; ≡表示 … if and only if …; (x) 表示 for all x; (∃x) 表示 there exists x. Also you may use ¬, ∧, →, ↔,∀, ∃

證明系統說明: See page 2.

(I) 翻譯 (8 points each)

The universe is the set of all humans.

- $S(x)$ = "x is a student,"
- $F(x)$ = "x is a faculty member,"
- $A(x, y)$ = "x has asked y a question,"
- *Lee* = "Professor Lee,"
- *Wang* = "Professor Wang."

(1) At least one student has never been asked a question by any faculty member.

(2) No student has ever asked Professor Wang a question.

(3) Every student has either asked Professor Lee a question or been asked a question by Professor Lee.

(4) Only those students who has asked Professor Wang a question has been asked a question by Professor Lee.

(5) There is a faculty member who has asked every other faculty member a question.

(II) 翻譯 (Replacing function symbols by relation symbols in relational predicate logic)

Consider the formula $S(x, w, f(x))$, where S is a 3-ary relation symbol and f is a unary function symbol. Since in relational predicate logic the function symbol f is not allowed, we are going to use a binary relation symbol R to replace the role of f. We define that $R(x, y)$ for $y = f(x)$ (here x, y are variables). Since there are two constraints for the function f : (i) for any x , there exists a y such that y and x have this relation (i.e. y is the output of x through f); (ii) such y is unique.

(6) Convert $S(x, w, f(x))$ into a formula without f by replacing it via R . (This translation needs to express both the existence and uniqueness of y.)(12 points)

(III) 舉反例證無效性 Prove the invalidity of the following argument by constructing a counterexample. (10 points)

(7) 1. (x)(∃y) Axy /∴(∃y)(x) Axy

(Warning: Examples from daily life may be problematic. A mathematical one is preferable.)

(IV)形式證明 Prove the following arguments. (You may use the systems given below or some similar system. However, semantic tableau system is less preferred and it may cause losing points (though *to write a tableau proof* is better than *to write nothing*.)

(8) (x)(Qx • ~Qx) /∴ (x) ~Px (10 points)

(9) (∃x)Rx ∨ (∃x)Sx /∴(∃x)(Rx ∨ Sx) (10 points)

(V)證明題 (6 points each)

What follows is part of the proof of Lindenbaum's theorem: If Σ is consistent, then there is a maximal consistent extension of Σ. (Recall that "Σ is consistent" means that one can not derive a contradiction from Σ by the proof system of first order logic.)

The proof is done as follows: First we enumerate all sentences in a list, say, $s_0, s_1, s_2, s_3, \ldots, s_n, \ldots$ and then define

$\Delta_0 = \Sigma$, for every $n \geq 0$, Δ_{n+1} is $\Delta_n \cup \{ s_n\}$ if it is consistent; else Δ_{n+1} is Δ_n.

(10) Prove that for any n, if Δ_n is consistent, then so is Δ_{n+1}.

(11) Prove that Δ_n is consistent for every $n \geq 0$.

(12) Prove that the union set of all Δ_n (that is, $\Delta_0 \cup \Delta_1 \cup \Delta_2 \cup \Delta_3 \cup \ldots \cup \Delta_n \cup \ldots$) is also consistent.

證明系統使用說明: 1. Most formal proof systems (Natural Deduction System, Gentzen's Sequent Calculus, the first system given below (or other similar systems), the second system given below (or other similar systems)) are acceptable, but <u>using ***semantic tableau system*** or other ***soundness-completeness unrecognizable proof systems*** may cause losing points</u> (*due to the undecidability of first order logic validity* (Church's theorem)).

2. <u>You are responsible for the **correct** way of using any of those systems or recalling meta-logical results (while not prohibited) in them</u>. That is, if you invalidly apply some inference rules (violating their constraints—though the constraints are not described below), it may cause losing points.

3. <u>It is OK to use different proof systems to solve different problems.</u> Within a problem, if one uses more than one system, it is subject to the grader's judgment.

You may use:

System I: This proof system contains implicational rules 1-8, equivalence rules 9-18, Conditional Proof (CP), Indirect Proof (IP), and 5 rules in predicate logic: one equivalence rule Quantifier Negation (QN), and 4 implicational rules: Universal Instantiation (UI), Existential Instantiation (EI), Universal Generalization (UG), Existential Generalization (EG). (<u>*You are supposed to know the rules and their constraints if you use them*</u>.)

1. MP $p \supset q$ / $p / \therefore q$

2. MT $p \supset q$ / $\sim q / \therefore \sim p$

3. DS $p \vee q$ / $\sim p / \therefore q$ | $p \vee q$ / $\sim q / \therefore p$

4. Simp $p \bullet q / \therefore p$ / $p \bullet q / \therefore q$

5. Conj p / $q / \therefore p \bullet q$

6. HS $p \supset q$ / $q \supset r / \therefore p \supset r$

7. Add $p / \therefore p \vee q$

8. CD $p \supset q$ / $r \supset s$ / $p \vee r / \therefore q \vee s$

9. DN $p :: \sim\sim p$

10. DeM $\sim (p \bullet q) :: \sim p \vee \sim q$ / $\sim (p \vee q) :: \sim p \bullet \sim q$

11. Comm $(p \vee q) :: (q \vee p)$ / $(p \bullet q) :: (q \bullet p)$

12. Assoc $[p \vee (q \vee r)] :: [(p \vee q) \vee r]$ / $[p \bullet (q \bullet r)] :: [(p \bullet q) \bullet r]$

13. Dist $[p \bullet (q \vee r)] :: [(p \bullet q) \vee (p \bullet r)]$ / $[p \vee (q \bullet r)] :: [(p \vee q) \bullet (p \vee r)]$

14. Contra $(p \supset q) :: (\sim q \supset \sim p)$

15. Impl $(p \supset q) :: \sim p \vee q$

16. Exp $[(p \bullet q) \supset r] :: [p \supset (q \supset r)]$

17. Taut $p :: (p \bullet p)$ / $p :: (p \vee p)$

18. Equiv $(p \equiv q) :: [(p \supset q) \bullet (q \supset p)]$ / $(p \equiv q) :: [(p \bullet q) \vee (\sim p \bullet \sim q)]$

System II: (Due to the offering of graduate logic course in CCU…)

Sentential axioms:

(A1) $\alpha \rightarrow (\beta \rightarrow \alpha)$

(A2) $[\alpha \rightarrow (\beta \rightarrow \gamma)] \rightarrow [(\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)]$

(A3) $(\neg \beta \rightarrow \neg \alpha) \rightarrow (\alpha \rightarrow \beta)$

Quantifier axioms:

(B1) $\forall x (\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \forall x \beta)$

where x is not a free variable of α.

(B2) $(\forall x \ \alpha(x)) \rightarrow \alpha(t)$

where the term t is free (or substitutable) for x in $\alpha(x)$.

Equality axioms: x, y are variables, $t(\ldots)$ is any term, α is any atomic formula, i is any integer such that $1 \le i \le n$.

(C1) $x = x$

(C2) $x = y \rightarrow (t(v_1 \ldots v_{i-1} x v_{i+1} \ldots v_n) = t(v_1 \ldots v_{i-1} y v_{i+1} \ldots v_n))$

(C3) $x = y \rightarrow (\alpha(v_1 \ldots v_{i-1} x v_{i+1} \ldots v_n) \leftrightarrow \alpha(v_1 \ldots v_{i-1} y v_{i+1} \ldots v_n))$

Two inference rules

(MP) α and $\alpha \rightarrow \beta$ infers β.

(Gen) α infers $\forall x \alpha$.

(And for fairness, <u>you are allowed to use Deduction Theorem</u> while using this system. <u>*You are supposed to know the rules and their constraints if you apply them*</u>.)