國立中正大學九十四學年度博士班招生考試試題

系所別：哲學所　　　　　　　　　　　　　　　　科目：邏輯

1. Answer the following questions (5 points each):
   a. Explain what is meant by “a truth-functional connective”, and give two examples (one monadic, one dyadic) showing that some ordinary sentential connectives are NOT truth-functional (explain why they are not).
   b. Let “p_1” and “p_2” be logical truths, “q_1” and “q_2” be logical falsity, “r” be a contingent truth and “s” be a contingent falsity. Which PAIRS of these sentences are logically equivalent? Which PAIRS of them are materially equivalent?
   c. Let $P = \{p_1, p_2\}$. Suppose S_1 is the set of models in which “p_1” is true, S_2 is the set of models in which “p_2” is true, and S’ is the set of models in which “q” is true. If $(S_1 \cap S_2) \subseteq S'$, then what is the consequence relation between P and “q”? If $(S_1 \cap S_2) = \varnothing$, then what is the consequence relation between P and “q”? If $(S_1 \cap S_2) \cap S' = \varnothing$, what is the consequence relation between “p” and “q”.

2. Let “~”, “$\rightarrow$” and “$\vee$” be defined as follows: “~p” is T if “p” is F or N, and is F if “p” is T. “$p \vee q$” is T if “p” or “q” is T, is F if both “p” and “q” are F, and is N otherwise. “$p \rightarrow q$” is T if both “p” and “q” are T, is F if “p” is T but “q” is F, and is N otherwise. Show that the following statements are true: (5 points each)
   a. “$p \vee \sim p$” is logically true, i.e., T in every model.
   b. {“$p \rightarrow q$”, “$q \rightarrow r$”} logically implies “$p \rightarrow r$”, i.e., no model can make “$p \rightarrow q$” and “$q \rightarrow r$” T, but “$p \rightarrow r$” not T. (DON’T draw a 27-line truth-table when doing this question; prove it directly.)
   c. “$p \rightarrow q$” does not logically imply “$\sim q \rightarrow \sim p$”, i.e., some model can make “$p \rightarrow q$” T but “$\sim q \rightarrow \sim p$” not T.

3. Think of the following first-order model M = <D, I> and a valuation function v, where D={a, b, c, d}, I(F)={a, b}, I(G)={b, c}, I(R)={<a, c>, <b, d>}, v(x)=a, v(y)=b, v(z)=c and v(w)=d. Decide which ones of the following claims are true. (2 points each)
   a. v satisfies $(x)[(Fx \,\&Gx) \supset Rxw]$
   b. v satisfies $(y)[(Fy \,\&Gy) \supset Rxz]$
   c. v satisfies $(\exists y)(Fy \,\&{\sim}Gy \,\&\, Ryw)$

d. v satisfies (∃w)(Fw &~Gw & Ryz)
e. v satisfies (x)Fx ⊃ Fz

4. Multiple choices (each question may have more than one “correct” answer): (5 points each)
   a. (　　　) An open formula ((1) may be satisfied by all sequences of objects; (2) may be satisfied by no sequence of objects; (3) may be satisfied by some sequences of objects yet not satisfied by others .)
   b. (　　　) A closed sentence ((1) may be satisfied by all sequences of objects; (2) may be satisfied by no sequence of objects; (3) may be satisfied by some sequences of objects yet not satisfied by others .)
   c. (　　　) A valid argument ((1) may have false conclusion; (2) may have inconsistent premises; (3) may have tautologous conclusion; (4) may be question-begging, i.e., may have a premise which is identical with the conclusion.)
   d. (　　　) ((1) (x)(Fx ⊃ Fx); (2) (x)Fx ⊃ Fy; (3) (x)Fx ≡ (y)Fy; (4) (∃x)x=y) is a valid first-order formula.

5. Specify a separate model for each of the following items showing in details that: (10 points each)
   a. It is false that (x)(Fx ⊃ (∃y)(Gy & Rxy)) ⊨ (∃y)(Gy & (x)(Fx ⊃ Rxy)).
   b. It is false that {(x)Rxx, (x)(y)(Rxy ⊃ Ryx)} ⊨ (x)(y)(z)[(Rxy & Ryz) ⊃ Rxz]

6. Translate the following argument into symbolic form: (10 points)
   Any horse can outrun any dog. Some greyhounds can outrun any rabbit. All greyhounds are dogs. Whatever x, y, z are, if x outruns y and y outruns z, then x outruns z. Therefore, any horse can outrun any rabbit. (Use the following key for translation. Hx: x is a horse; Dx: x is a dog; Oxy, x can overrun y; Gx: x is a greyhound; Rx: x is a rabbit.)

7. Prove by whatever proof-theoretical method you know that the argument in 6 is valid: (10 points)