# 國立中正大學九十四學年度博士班招生考試試題

系所別：哲學所　　　　　　　　　　　　　　科目：邏輯

1. Answer the following questions (5 points each):
   a. Explain what is meant by "a truth-functional connective", and give two examples (one monadic, one dyadic) showing that some ordinary sentential connectives are NOT truth-functional (explain why they are not).
   b. Let "$p_1$" and "$p_2$" be logical truths, "$q_1$" and "$q_2$" be logical falsity, "r" be a contingent truth and "s" be a contingent falsity. Which PAIRS of these sentences are logically equivalent? Which PAIRS of them are materially equivalent?
   c. Let $P = \{p_1, p_2\}$. Suppose $S_1$ is the set of models in which "$p_1$" is true, $S_2$ is the set of models in which "$p_2$" is true, and S' is the set of models in which "q" is true. If $(S_1 \cap S_2) \subseteq S'$, then what is the consequence relation between P and "q"? If $(S_1 \cap S_2) = \varnothing$, then what is the consequence relation between P and "q"? If $(S_1 \cap S_2) \cap S' = \varnothing$, what is the consequence relation between "p" and "q".

2. Let "~", "$\rightarrow$" and "$\vee$" be defined as follows: "~p" is T if "p" is F or N, and is F if "p" is T. "$p \vee q$" is T if "p" or "q" is T, is F if both "p" and "q" are F, and is N otherwise. "$p \rightarrow q$" is T if both "p" and "q" are T, is F if "p" is T but "q" is F, and is N otherwise. Show that the following statements are true: (5 points each)
   a. "$p \vee \sim p$" is logically true, i.e., T in every model.
   b. {"$p \rightarrow q$", "$q \rightarrow r$"} logically implies "$p \rightarrow r$", i.e., no model can make "$p \rightarrow q$" and "$q \rightarrow r$" T, but "$p \rightarrow r$" not T. (DON'T draw a 27-line truth-table when doing this question; prove it directly.)
   c. "$p \rightarrow q$" does not logically imply "$\sim q \rightarrow \sim p$", i.e., some model can make "$p \rightarrow q$" T but "$\sim q \rightarrow \sim p$" not T.

3. Think of the following first-order model M = <D, I> and a valuation function v, where D={a, b, c, d}, I(F)={a, b}, I(G)={b, c}, I(R)={<a, c>, <b, d>}, v(x)=a, v(y)=b, v(z)=c and v(w)=d. Decide which ones of the following claims are true. (2 points each)
   a. v satisfies $(x)[(Fx \& Gx) \supset Rxw]$
   b. v satisfies $(y)[(Fy \& Gy) \supset Rxz]$
   c. v satisfies $(\exists y)(Fy \& \sim Gy \& Ryw)$

d. v satisfies $(\exists w)(Fw \,\&\, {\sim}Gw \,\&\, Ryz)$
e. v satisfies $(x)Fx \supset Fz$

4. Multiple choices (each question may have more than one “correct” answer): (5 points each)
   a. （　　　）An open formula（(1) may be satisfied by all sequences of objects; (2) may be satisfied by no sequence of objects; (3) may be satisfied by some sequences of objects yet not satisfied by others .）
   b. （　　　）A closed sentence（(1) may be satisfied by all sequences of objects; (2) may be satisfied by no sequence of objects; (3) may be satisfied by some sequences of objects yet not satisfied by others .）
   c. （　　　）A valid argument （(1) may have false conclusion; (2) may have inconsistent premises; (3) may have tautologous conclusion; (4) may be question-begging, i.e., may have a premise which is identical with the conclusion.）
   d. （　　　）（(1) $(x)(Fx \supset Fx)$; (2) $(x)Fx \supset Fy$; (3) $(x)Fx \equiv (y)Fy$; (4) $(\exists x)x=y$）is a valid first-order formula.

5. Specify a separate model for each of the following items showing in details that: (10 points each)
   a. It is false that $(x)(Fx \supset (\exists y)(Gy \,\&\, Rxy)) \vDash (\exists y)(Gy \,\&\, (x)(Fx \supset Rxy))$.
   b. It is false that $\{(x)Rxx, (x)(y)(Rxy \supset Ryx)\} \vDash (x)(y)(z)[(Rxy \,\&\, Ryz) \supset Rxz]$

6. Translate the following argument into symbolic form: (10 points)
   Any horse can outrun any dog. Some greyhounds can outrun any rabbit. All greyhounds are dogs. Whatever x, y, z are, if x outruns y and y outruns z, then x outruns z. Therefore, any horse can outrun any rabbit. (Use the following key for translation. Hx: x is a horse; Dx: x is a dog; Oxy, x can overrun y; Gx: x is a greyhound; Rx: x is a rabbit.)

7. Prove by whatever proof-theoretical method you know that the argument in 6 is valid: (10 points)