

國立中正大學 碩士班甄試 邏輯試題 2007/11/23

In this test,

“ $\neg$ ” means “not”,

“ $\wedge$ ” means “and”,

“ $\vee$ ” means “or”,

“ $\rightarrow$ ” means “if...then...”,

“ $\leftrightarrow$ ” means “if and only if”,

“ $\forall x$ ” means “for all x” and

“ $\exists x$ ” means “for some x”.

I. Let “ $Lxy$ ” stand for “x loves y”,  
“ $Hxy$ ” stand for “x hates y” and  
“ $Px$ ” stand for “x is a philosopher”.

Please symbolize the following two sentences. (15 points each)

- (1) There are exactly two philosophers who love each other.
- (2) Some philosophers hate everyone unless someone loves them.

II. Please circle the tautologies from the following formulas. (15 points)

$$A \rightarrow [(B \wedge C) \rightarrow A]$$

$$(A \wedge B) \vee [(\neg A \vee \neg B) \vee (C \wedge \neg C)]$$

$$(A \rightarrow A) \rightarrow A$$

$$(A \rightarrow B) \rightarrow [(\neg B \wedge C) \rightarrow \neg A]$$

$$\neg [A \wedge (\neg A \vee B)]$$

$$[(A \wedge B) \rightarrow C] \rightarrow [B \rightarrow (A \rightarrow (D \rightarrow C))]$$

III. Please give counterexamples to the following two **invalid** arguments. (15 points each)

$$(1) \forall x \exists y Lxy / \therefore \exists y \forall x Lxy$$

$$(2) \forall x Px \rightarrow \exists y Ry / \therefore \forall x (Px \rightarrow \exists y Ry)$$

IV. Please prove the following **valid** argument. (You may use the system on the next page. But virtually all formal proof systems are acceptable; just make your proof as clear as possible). (25 points)

$$\forall x (Px \rightarrow (Qx \vee Rx)), \forall x \neg R(x) / \therefore \forall x (Px \rightarrow Qx)$$