In this test,
"$\neg$" means "not",
"$\wedge$" means "and",
"$\vee$" means "or",
"$\rightarrow$" means "if...then...",
"$\leftrightarrow$" means "if and only if",
"$\forall x$" means "for all x", and
"$\exists x$" means "for some x".

I. True or False

Answer each of the following questions simply by writing **True** or **False. (24 points; 3 points each)**

1. $(A \wedge E) \rightarrow [\neg(A \leftrightarrow F) \leftrightarrow (\neg E \leftrightarrow F)]$ is a tautology.
2. If $R(a, b)$ logically implies $R(b, a)$, then $a=b$.
3. If α, β and γ are inconsistent, then $(\beta \leftrightarrow \gamma) \leftrightarrow \alpha$ cannot be true.
4. $\forall x \forall y \alpha(x, y)$ is satisfiable if and only if $\alpha(c, c)$ is satisfiable, where c is a constant symbol which does not occur in $\alpha(x, y)$.
5. $\forall x \exists y \forall z(P(x, z) \rightarrow Q(y))$ is logically equivalent to $\exists x \exists z P(x, z) \rightarrow \exists y Q(y)$.
6. Suppose S is a set of infinitely many sentences and T is a finite subset of S. Then there must be a theorem of S which is not provable from T.
7. Suppose S and T are two consistent sets of sentences such that $S \cup T$ is inconsistent. Then for some α in T, S proves $\neg\alpha$.
8. No finite model can satisfy $\forall x \exists y(x \neq y \wedge R(x, y)) \wedge \forall x \forall y \forall z(R(x, y) \rightarrow (R(y, z) \rightarrow R(x, z))$.

II. Symbolize the following two sentences. **(20 points; 10 points each)**

(1) 每一對已生育的夫妻每月可領取一萬元的補助，但月收入若已達到十二萬元(含)以上則不得領取。

(2) At least two philosophers are talking nonsense unless everyone who attends the meeting is from Mars.

III. Define a sentential connective + as follows. A+B is true if and only if exactly one of A and B is true (that is, A is true and B is false or A is false and B is true). Show that $\rightarrow$ is not definable by using $\neg$, $\leftrightarrow$ and + (that is, no sentence in the language using A, B, $\neg$, $\leftrightarrow$ and + is equivalent to $A \rightarrow B$). **(10 points)**

IV. For each of the following arguments, prove it or refute it by giving a counterexample. **(46 points)**

(1) 前提 1：並非所有的聰明人都是哲學家。 前提 2：所有的哲學家都是苦命人。 因此，有些聰明

人不是苦命人。**(6 points)**

(2) $\forall x(Rx \leftrightarrow Qx), \exists x(\neg(Px \leftrightarrow Qx) \leftrightarrow Rx), \forall y(S(x, y) \rightarrow Rx)$ /∴ $\forall x((\exists yRy \wedge \exists yQy) \rightarrow Px) \rightarrow \neg S(x, b)$ **(10 points)**

(3) $\neg(A \leftrightarrow B) \leftrightarrow C, B \rightarrow (C \rightarrow D)$ /∴ $(D \wedge A) \vee (B \leftrightarrow C)$ **(10 points)**

(4) /∴ $\exists z \forall y \exists x((P(z) \rightarrow R(y)) \rightarrow (P(x) \rightarrow R(x)))$ **(10 points)**

(5) $\forall x \exists y R(x, y), \forall x \forall y(R(x, y) \rightarrow \exists z(z \neq x \wedge z \neq y \wedge R(x, z) \wedge R(z, y))$ /∴ $\exists x \exists y \exists z(x \neq y \wedge x \neq z \wedge y \neq z \wedge R(x, y) \wedge R(y, z))$ **(10 points)**