國立中正大學 2002/12/22 甄試邏輯試題

符號: ~ 表示 not; • 表示 and; ∨ 表示 or (inclusive or); ⊃表示 if… then …;
≡表示 … if and only if …

(I) Lxy means "x loves y", Mx means "x is a man", Wx means "x is a woman", and the domain of discourse is all human.

Symbolizing the following sentences

(1) Everyone is either a man or a woman. (12 points)

(2) No one loves himself(herself). (12 points)

(3) Not every man loves every woman. (12 points)

(4) Anyone who loves a woman loves everyone. (14 points)

(II) Prove the invalidity of the following argument by constructing a counterexample. (15 points)

(5) $(x)(\exists y)\,Exy \quad /\therefore (\exists x)(y)\,Exy$

Hint for (5): Consider domain $=\{a,b\}$ or try some possible meaning of E.

(III) Prove the following arguments. (You may use the system given in the next page **(all rules are applicable)** or some similar system. However, semantic tableau system is less preferred and it may cause losing some points (though it is better to *write a tableau proof* than to *write nothing*.)

(6) 1. $(x)\,Px$

2. $(\exists x)(Qx \bullet \sim Px) \; /\therefore \; (x)\,Qx$ (18 points)

(7) 1. $(\exists x)(y)\,Exy /\therefore (y)(\exists x)\,Exy$ (17 points)

說明: Almost all formal proof systems (especially systems similar to the following one) are essentially acceptable, but using semantic tableau system may cause losing certain points.

You may use the following system, which contains implicational rules 1-8, equivalence rules 8-18, Conditional Proof (CP), Indirect Proof (IP), and 5 rules in predicate logic: one equivalence rule Quantifier Negation (QN), and 4 implicational rules: Universal Instantiation (UI), Existential Instantiation (EI), Universal Generalization (UG), Existential Generalization (EG).

1. MP $p \supset q$, $p / \therefore q$

2. MT $p \supset q$, $\sim q / \therefore \sim p$

3. DS $p \vee q$, $\sim p / \therefore q$ | $p \vee q$, $\sim q / \therefore p$

4. Simp $p \bullet q / \therefore p$, $p \bullet q / \therefore q$

5. Conj p, $q / \therefore p \bullet q$

6. HS $p \supset q$, $q \supset r / \therefore p \supset r$

7. Add $p / \therefore p \vee q$

8. CD $p \supset q$, $r \supset s$, $p \vee r / \therefore q \vee s$

9. DN $p :: \sim\sim p$

10. DeM $\sim(p \bullet q) :: \sim p \vee \sim q$; $\sim(p \vee q) :: \sim p \bullet \sim q$

11. Comm $(p \vee q) :: (q \vee p)$; $(p \bullet q) :: (q \bullet p)$

12. Assoc $[p \vee (q \vee r)] :: [(p \vee q) \vee r]$; $[p \bullet (q \bullet r)] :: [(p \bullet q) \bullet r]$

13. Dist $[p \bullet (q \vee r)] :: [(p \bullet q) \vee (p \bullet r)]$; $[p \vee (q \bullet r)] :: [(p \vee q) \bullet (p \vee r)]$

14. Contra $(p \supset q) :: (\sim q \supset \sim p)$

15. Impl $(p \supset q) :: \sim p \vee q$

16. Exp $[(p \bullet q) \supset r] :: [p \supset (q \supset r)]$

17. Taut $p :: (p \bullet p)$; $p :: (p \vee p)$

18. Equiv $(p \equiv q) :: [(p \supset q) \bullet (q \supset p)]$; $(p \equiv q) :: [(p \bullet q) \vee (\sim p \bullet \sim q)]$