In this test,

"$\neg$" means "not",

"$\wedge$" means "and",

"$\vee$" means "or",

"$\rightarrow$" means "if... then..." ,

"$\leftrightarrow$" means "if and only if",

"$\forall x$" means "for all x" and

"$\exists x$" means "for some x".

I. True or False

Please answer each of the following questions by writing True or False. No explanation is needed. (30 points; 3 points each)

1. If the conclusion of an argument is a contradiction, then that argument is invalid.
2. $\exists x\alpha(x)\rightarrow\alpha(x)$ and $\exists y\alpha(y)\rightarrow\alpha(y)$ are logically equivalent.
3. If A and B are inconsistent, then either A or B is not a tautology.
4. $\neg(A \leftrightarrow B) \leftrightarrow (B \leftrightarrow C)$ is logically equivalent to $A \leftrightarrow \neg C$.
5. If α logically implies β and β is not a logical truth, then β cannot contain any free variable which does not occur free in α.
6. "All human beings are mortal. All heroes are human beings. Therefore, some heroes are mortal." This argument is valid.
7. If α logically implies β, then $\gamma \vee \alpha$ logically implies $(\gamma\rightarrow\alpha)\rightarrow\beta$.
8. $\forall x(\mathrm{P}x \rightarrow \exists x\mathrm{Q}x)$ and $\exists x\mathrm{P}x \rightarrow \exists x\mathrm{Q}x$ are logically equivalent.
9. $\exists y(\mathrm{R}(x,y) \rightarrow \exists u\forall v\mathrm{R}(u,v))$ is a logical truth.
10. For any logical theory T and any sentence α in T's language, it must be the case that either T logically implies α or T logically implies $\neg\alpha$.

II. Symbolization

Let "Sx" stand for "x is a student", "Px" for "x is a philosopher", "Wx" for "x passes the test", "Ixy" for "x interviews y" and "Lxy" for "x loves y". Please symbolize the following two sentences. (20 points; 10 points each)

1. There are at least two students who pass the test and who are interviewed by exactly three philosophers.
2. Some student who passes the test loves all philosophers, unless that student is interviewed by a philosopher who doesn't love her/him.

III. For each of the following arguments, give a proof or a counterexample.
You can use any kind of method with which you are familiar; just try to make your proofs or counterexamples as clear as possible. (40 points; 10 points each)

1. $A \leftrightarrow B$, $(A \vee ((E \rightarrow F) \wedge A)) \rightarrow C$, $\neg C \vee D$ /∴ $\neg(B \wedge \neg D)$
2. $A \rightarrow E$, $(B \vee C) \rightarrow D$, $\neg(E \wedge (B \rightarrow \neg C))$ /∴ $(D \rightarrow A) \rightarrow (B \leftrightarrow C)$
3. $\forall x(Px \rightarrow \forall y \exists z Ryz)$, $\forall x \neg(Px \leftrightarrow (Qx \leftrightarrow Sx))$, $\forall x(Qx \vee Sx)$ /∴ $\forall y(\exists x \neg Sx \rightarrow \exists x Ryx)$
4. $\forall x \forall y \forall z((Rxy \wedge Ryz) \rightarrow Rxz)$, $\forall x \neg Rxx$, $\forall x \exists y(Rxy \wedge \neg \exists z(Rxz \wedge Rzy))$
/∴ $\forall x \exists y(Rxy \wedge \forall z(z=y \vee (Rxz \rightarrow Ryz)))$

IV. Please explain why we cannot translate "all P are Q" into "$\forall x(Px \wedge Qx)$" and "some P are Q" into "$\exists x(Px \rightarrow Qx)$". (10 points)