- In the following questions, we will use (1) $\sim$ as "negation, (2) $\wedge$ as "conjunction", (3)$\vee$ as "or", (4) $\to$ as "implication", and (5)$\leftrightarrow$ as "equivalence", (6) (x) as "for all x", (7) $(\exists x)$ as "for some x", and $\therefore$ for "therefore" in an argument.
- If you want to answer the questions with the symbols you are familiar with, explicitly state the intended meaning of the symbols you are using.
- When proving, you can use the proof system that you are familiar with, but specify the source of your proof system (from which book and who is the author).

1. Translate the following English sentences into well-formed formulas of sentential logic, with English alphabets standing for *atomic sentences* - that is, those sentences that are not built up out of other sentences. (10pts)
   (a) John walks home and Paul reads a book.
   (b) Paul doesn't ride a bike.
   (c) It is not the case that if Mary joins the party, then Petty does so.
   (d) Either Mary or Petty is absent.

2. Use truth table method to show the validity of the following arguments. (5pt)
   1. $(A \to B) \vee C$ 2. $\sim B$ $\therefore A \to C$.

3. Prove that the following argument is a valid argument (no semantic method).(20pt)
   (a) 1. $A \vee (B \wedge C)$ 2. $\sim C$ $\therefore A$.
   (b) 1. $(A \wedge B) \vee C$ 2. $(A \wedge B) \to (E \to A)$ 3. $C \to D$ $\therefore (E \to A) \vee D$.

4. Translate the following sentences into well-formed formulas in first-order logic. (15pts)
   (a) Not all candies taste sweet.
   (b) If every one is rational, then Socrates is rational.
   (c) Some boy likes some girl who likes herself.

5. Prove the validity of the following argument. (20pt)
   (a) 1. $\sim (x)Px / \therefore (\exists x)(Px \to Qx)$
   (b) 1. $(x) \sim Px / \therefore (y)(Qy \to \sim Py)$

6. Show that the following argument is invalid (10pt)
   1. $(x)(Px \to Qx)$,
   2. $(\exists x) \sim Px / \therefore (\exists x) \sim Qx$.

7. Prove the following formula is universally valid: (10pt)
   $(x)Px \to (\exists x)Px$

8. Let A, B be sets. If $A \subseteq B$, $A \cup B = B$. (10pt)