- In the following questions, we will use (1) $\neg$ as *negation*, (2) $\wedge$ as *conjunction*, (3) $\vee$ as *or*, (4) $\rightarrow$ as *implication*, and (5) $\leftrightarrow$ as *equivalence*, (6) (x) as *or all* x, (7) $(\exists x)$ as *for some* x, and $\therefore$ for *therefore* in an argument.
- If you want to answer the questions with the symbols you are familiar with, explicitly state the intended meaning of the symbols you are using.
- When proving, you can use the proof system that you are familiar with, but specify the source of your proof system (from which book and who is the author).

1. Use your own symbolism to translate the following sentences into logical formulas (in propositional or predicate logic), and reveal the logical structure of the sentences as much as possible. Remember to indicate the meaning of your symbolism: (30pt)
   a. John is a friend of Mary.
   b. Either Peter is a friend of Mary, or Mark is a friend of Mary.
   c. Everybody is a friend of Peter.
   d. If someone is a friend of Mary, then Mary is a friend of someone.
   e. Any sets that have the same members are equal.
   f. There is no barber who shaves precisely those men who do not shave themselves.

2. In the following questions, use truth table method to show if the pair of sentences in the question are logically equivalent. ($\phi, \psi, \theta, \gamma$ are formulas) (20pt)
   a. $(\phi \vee (\psi \wedge \theta)), ((\phi \vee \psi) \wedge (\phi \vee \theta))$
   b. $\neg(\phi \rightarrow \psi), (\neg\phi \rightarrow \neg\psi)$

3. Prove that the following argument is a valid argument (no semantic method) (30pt)
   a. 1. $(\phi \rightarrow \psi)$
      2. $(\theta \rightarrow \phi)$
      3. $(\theta \vee (\psi \wedge \gamma)) / \therefore \psi$
   b. 1. $(x)(H(x) \rightarrow K(x))$
      2. $(\exists x)H(x) \vee (\exists x)K(x)$ / $\therefore$ $(\exists x)K(x)$

4. Show that the following argument is invalid (10pt)
   1. $(\exists x)(P(x) \wedge \neg Q(x))$
   2. $(x)(R(x) \rightarrow P(x))$
   / $\therefore$ $(\exists x)(R(x) \wedge \neg Q(x))$

5. Prove that if $A_1, \ldots, A_n$ and $B_1, \ldots, B_n$ are sets such that $A_i \subseteq B_i$ for $i = 1,2,\ldots,n$, then $\bigcup_{i=1}^{n} A_i \subseteq \bigcup_{i=1}^{n} B_i$. (10pt)