In this test,
"$\neg$" means "not",
"$\wedge$" means "and",
"$\vee$" means "or",
"$\rightarrow$" means "if...then...",
"$\leftrightarrow$" means "if and only if",
"$\forall x$" means "for all x", and
"$\exists x$" means "for some x".

I. True or False. Please answer each of the following questions by writing **True** or **False.** No explanation is necessary. **(30 points; 3 points each)**

1. An argument can have true premises and a true conclusion and yet may not be logically valid.
2. $\forall x(P(x)\leftrightarrow R(x))$ is logically equivalent to $\forall xP(x)\leftrightarrow\forall xR(x)$.
3. Suppose x does not occur free in φ (that is, x is not a free variable in φ). Then we can infer $\forall x\varphi$ from φ.
4. If $(P\vee R)\wedge S$ implies Q and S does not imply Q, then P implies $P\rightarrow Q$ and R implies $R\rightarrow Q$.
5. If A and B are inconsistent, then $\neg A$ implies B.
6. If some P are Q and some Q are S, then some S are not P.
7. If $\exists x\exists y(Px \wedge Py)$ is true in a model, then the domain of that model must contain at least two members.
8. Let R be a binary predicate. Then a=b whenever $R(a, b)\leftrightarrow R(b, a)$ is the case.
9. Suppose only one of the following two sentences is true: (1) If Kant is right, then pigs can fly; (2) If Kant is not right, then pigs can fly. Hence it is true that if pigs can fly, then Kant is right (Assume that the conditional here is material).
10. If every woman loves some man, then every woman loves some man who loves her. Hence no woman can love some man who does not love her.

II. Please give counterexamples to the following two **invalid** arguments. **(20 points; 10 points each)**

(1) $\forall x\neg R(x, c)$, $\forall x(x\neq c\rightarrow\exists y(R(y, x)\wedge\forall z(R(z, x)\rightarrow y=z)))$,
$\forall x\exists y(R(x, y)\wedge\forall z(R(x, z)\rightarrow y=z))$, $\forall x(Q(c, x)\leftrightarrow\exists y(Q(c, y)\wedge R(y, x)))$
$/ \therefore \forall xQ(c, x)$

(2) $(\exists xP(x)\wedge\exists xR(x))\wedge(\forall xP(x)\leftrightarrow\forall xR(x))$ /∴ $\exists x(P(x)\wedge R(x))$

III. Please symbolize the following two sentences. **(20 points; 10 points each)**

(1) Any unmarried man who is over forty is obsessed with exactly two young women each of whom is loved by someone who hates every man who loves her.

(2) 可憐之人必有可恨之處。

IV. Please prove the following two **valid** arguments. (You may use the system on the next page. But virtually all formal proof systems are acceptable; just try to make your proofs as clear as possible). **(30 points; 15 points each)**

(1) $\{[(A\wedge\neg B)\vee(B\wedge\neg C)\vee(C\wedge\neg A)]\rightarrow[(A\wedge B\wedge C\wedge D)\rightarrow(E\leftrightarrow F)]\}\leftrightarrow((J\vee K)\rightarrow(G\wedge C))$, $[((B\leftrightarrow E)\leftrightarrow(C\leftrightarrow D))\leftrightarrow((A\leftrightarrow B)\leftrightarrow\neg(D\leftrightarrow E))]$ /∴ $A\rightarrow(J\rightarrow F)$

(2) $\forall x(P(x)\rightarrow\exists y(Q(y)\vee R(x)))$, $\forall x\neg R(x)$ /∴ $\forall x(P(x)\rightarrow\exists yQ(y))$

You may use the following system, which contains implicational rules **1-8**, equivalence rules **9-18**, Conditional Proof(**CP**), Indirect Proof(**IP**), and 5 rules in predicate logic: one equivalence rule Quantifier Negation (**QN**), and 4 implicational rules: Universal Instantiation(**UI**), Existential Instantiation(**EI**), Universal Generalization(**UG**), Existential Generalization(**EG**). In the following, "$A :: B$" *means* "$A / \therefore B$ *and* $B / \therefore A$".

1.MP:
$p \to q$
$p / \therefore q$

2.MT:
$p \to q$
$\neg q / \therefore \neg p$

3.DS:
$p \vee q$
$\neg p / \therefore q$

$p \vee q$
$\neg q / \therefore p$

4.Simp:
$p \wedge q / \therefore p$
$p \wedge q / \therefore q$

5.Conj:
p
$q / \therefore p \wedge q$

6.HS:
$p \to q$
$q \to r / \therefore p \to r$

7.Add:
$p / \therefore p \vee q$

8.CD:
$p \to q$
$r \to s$
$p \vee r / \therefore q \vee s$

9.DN : $p :: \neg\neg p$

10.DeM : $\neg(p \wedge q) :: \neg p \vee \neg q$
$\neg(p \vee q) :: \neg p \wedge \neg q$

11.$Comm$: $(p \vee q) :: (q \vee p)$
$(p \wedge q) :: (q \wedge p)$

12.$Assoc$: $[p \vee (q \vee r)] :: [(p \vee q) \vee r]$
$[p \wedge (q \wedge r)] :: [(p \wedge q) \wedge r]$

13.$Dist$: $[p \wedge (q \vee r)] :: [(p \wedge q)] \vee (p \wedge r)]$
$[p \vee (q \wedge r)] :: [(p \vee q) \wedge (p \vee r)]$

14.$Contra$: $(p \to q) :: (\neg q \to \neg p)$

15.Impl: $(p \to q) :: \neg p \vee q$

16.Exp : $[(p \wedge q) \to r] :: [p \to (q \to r)]$

17.$Taut$: $p :: (p \wedge p)$
$p :: (p \vee p)$

18.$Equiv$: $(p \leftrightarrow q) :: [(p \to q) \wedge (q \to p)]$
$(p \leftrightarrow q) :: [(p \wedge q) \vee (\neg p \wedge \neg q)]$