國立中正大學 碩士班甄試邏輯試題 2012/11/16

In this test,

"¬" means "not",

"∧" means "and",

"∨" means "or",

"→" means "if...then...",

"↔" means "if and only if",

"∀x" means "for all x", and

"∃x" means "for some x".

I.   True or False. Please answer each of the following questions simply by writing True or False. (40 points; 4 points each)

1.   If the conclusion of an argument is a contradiction, then that argument is invalid.

2.   If ∃x ∃y (Px ∧ Py) is true in a model, then the domain of that model must contain at least two members.

3.   ∀x (Px ∨ Qx) is logically equivalent to ∀x Px ∨ ∀x Qx.

4.   Two different constants must have different interpretations.

5.   P and Q are inconsistent if and only if P and Q have different truth values.

6.   A ↔ ¬ ((B ↔ (¬ (A ↔ C) ↔ C)) ↔ B) is a tautology.

7.   Predicate symbols are logical symbols.

8.   If an argument is valid, then it is sound.

9.   Suppose A is contingent. If A and B are inconsistent and A and C are inconsistent, then B and C must be inconsistent.

10.   If A → (B → (C → D)) is not a tautology, then D is not a tautology.

II.   Let "Lxy" stand for "x loves y",
"Hxy" stand for "x hates y" and
"Px" stand for "x is a philosopher".
Please symbolize the following sentence. (20 points)

1.   All philosophers hate at most one person who is not a philosopher and who loves exactly one philosopher.

III. Please prove the following two valid arguments (Any proof system is acceptable; just try to make your proofs as clear as possible). (40 points; 20 points each)

1.   (¬ (A ↔ B) ∨ (A ∧ B)) → (C ∨ (A ∧ C)) /∴ (A ∨ B) → C

2.   ¬ (∀x ∃y (¬ Sxy ↔ Sxx) ∨ ∃y ∀x Pxy) ∨ (¬∀x ∃y (Sxy ↔ ¬ Sxx) ∧ ∀x ∃y Pxy) /∴∃x (∃y Sxy → ∀y Sxy)