國立中正大學　碩士班甄試　邏輯試題 2014/11/14

In this test,
"$\neg$" means "not",
"$\wedge$" means "and",
"$\vee$" means "or",
"$\rightarrow$" means "if... then...",
"$\leftrightarrow$" means "if and only if",
"$\forall x$" means "for all x" and
"$\exists x$" means "for some x".

I. True or False

Please answer each question by writing True or False. No explanation is needed. (30 points; 3 points each)

1. $(A \leftrightarrow B) \wedge (B \leftrightarrow C)$ is logically equivalent to $A \leftrightarrow (B \leftrightarrow C)$
2. If $A \wedge B$ is a contradiction, $B \wedge C$ is a contradiction and B is not a contradiction, then A is logically equivalent to C.
3. If A implies B, then A and C together also imply B.
4. $(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$ is a tautology.
5. If $A \rightarrow B$ is a tautology and A is not a contradiction, then B is a tautology.
6. In the formula $\exists x \forall x Px$, the "x" occurring in "Px" is bounded by "$\exists x$".
7. $\forall x Px$ is logically equivalent to $\forall y Py$.
8. Px is logically equivalent to Py.
9. The interpretations of two different constants cannot be the same.
10. $\forall x \exists y Pxy$ is logically equivalent to $\exists y \forall x Pxy$

II. Symbolization

Let ''Px" stand for "x is a philosopher", "Lxy" stand for "x loves y" and "Hxy" stand for "x hates y". Please symbolize the following two sentences. (30 points; 15 points for each)

1. All philosophers love someone who is not a philosopher, unless someone who is not a philosopher hates all philosophers.
2. If there are exactly two philosophers loved by all philosophers, then some philosopher hates at most two people who are not philosophers.

III. Please give a counterexample to the following invalid argument. (20 points)

1. $\forall x(Px \vee Qx) / \therefore \forall x Px \vee \forall x Qx$

IV. Please prove the following valid argument.

Virtually all formal proof systems are acceptable; just make your proof as clear as possible. (20 points)

1. $\exists x \forall y (Sxy \leftrightarrow Sxx) / \therefore \exists x(\forall y Sxy \vee \forall y \neg Sxy)$