臺灣綜合大學系統 109 學年度學士班轉學生聯合招生考試試題

| 科目名稱 | 邏輯 | 類組代碼 | D23 |
| --- | --- | --- | --- |
| | | 科目碼 | D2392 |

※本項考試依簡章規定所有考科均「不可」使用計算機。 本科試題共計 2 頁

In this test,
"¬" means "not",
"∧" means "and",
"∨" means "or",
"→" means "if...then...",
"↔" means "if and only if",
"∀x" means "for all x" and
"∃x" means "for some x".

**I. True or False**

Please answer each question by writing **True** or **False.** No explanation is needed. **(20 points; 2 points each)**

1. Two different predicates must have different interpretations (extensions).
2. $P(a,b,a) \leftrightarrow P(b,a,b)$ logically implies $a = b$.
3. If $\exists x\exists y(P(x)\wedge P(y))$ is true in a model, then the domain of that model must contain at least two members.
4. $\forall x(P(x)\rightarrow Q(x))$ is logically equivalent to $\forall xP(x)\rightarrow\forall xQ(x)$.
5. $[(A\wedge B)\rightarrow C]\rightarrow[B\rightarrow(A\rightarrow(D\rightarrow C))]$ is a tautology.
6. If every woman loves some man, then every woman loves some man who loves her. Hence no woman can love some man who does not love her.
7. P logically implies Q if and only if P logically implies $P\rightarrow Q$.
8. Suppose A is not a contradiction. If A and B are inconsistent and A and C are inconsistent, then B and C must be inconsistent.
9. Every premise in a valid argument must be true.
10. Suppose P and S are consistent. Then P cannot logically imply Q if S and Q are inconsistent.

**II. Symbolization**

Let "Lxy" stand for "x loves y",
"Hxy" stand for "x hates y",
"Px" stand for "x is a philosopher"
and "Kx" stand for "x is a king".

Please symbolize the following two sentences. **(20 points; 10 points each)**

(1) No philosopher can be a king unless some king hates another king.
(2) There are exactly two philosophers who love each other.

# 臺灣綜合大學系統 109 學年度學士班轉學生聯合招生考試試題

| 科目名稱 | 邏輯 | 類組代碼 | D23 |
|---|---|---|---|
| | | 科目碼 | D2392 |

※本項考試依簡章規定所有考科均「不可」使用計算機。　　本科試題共計 2 頁

**III.** Please give counterexamples to the following three **invalid** arguments. **(30 points; 10 points each)**

(1) $\neg(A \leftrightarrow B) \leftrightarrow C,\ B \rightarrow (C \rightarrow D)\ /\therefore\ (D \wedge A) \vee (B \leftrightarrow C)$
(2) $\exists x P(x) \wedge \exists x R(x),\ \forall x P(x) \leftrightarrow \forall x R(x)\ /\therefore\ \exists x(P(x) \wedge R(x))$
(3) $\exists x \forall y R(x, y)\ /\therefore\ \exists y R(x, y)$

**IV.** Please prove the following two **valid** arguments (all formal proof systems are acceptable; just make your proofs as clear as possible). **(30 points; 15 points each)**

(1) $A \vee \neg B,\ (B \rightarrow C) \rightarrow D\ /\therefore\ (A \vee D) \wedge (E \vee (\neg D \rightarrow A))$
(2) $\forall x(Rx \leftrightarrow Qx),\ \exists x(\neg(Px \leftrightarrow Qx) \leftrightarrow Rx)\ /\therefore\ \forall x((\exists y Ry \wedge \exists y Qy) \rightarrow Px) \rightarrow \forall x \neg Rx$