國立中正大學 2003/6/15 博士班入學邏輯試題 (11 problems, total 100 points)

符號說明: ~ 表示 not; • 表示 and; ∨ 表示 or (inclusive or); ⊃表示 if… then …; ≡表示 … if and only if …; (x) 表示 for all x; (∃x) 表示 there exists x. Also you may use ¬, ∧, →, ↔,∀, ∃

證明系統說明: See page 2.

(I) 翻譯 (Literacy in Logic)

Symbolizing the following sentences. Here Lxy means "x loves y", Mx means "x is a man", Wx means "x is a woman", and the domain of discourse is all human.

(1) Not every woman loves every man. (5 points)

(2) Anyone who loves a woman loves someone. (5 points)

(3) Any man who is loved by everyone is loved by every woman. (10 points)

(II) 翻譯 (More literacy in Logic involving a little mathematics)

Symbolizing the following sentences

(4) For any sets x, y, if every element of x is an element of y, and every element of y is an element of x, then x equals y. (10 points)

(**NOTE:** the domain of discourse is all sets, and you are allowed to use ∈ (belongs to), = (equals), please try not to use ⊆.)

(5) If P(0) is true, and for any (natural number) x P(x) is true implies P(x+1) is true, then for any (natural number) x P(x) is true. (10 points)

(**NOTE:** the domain of discourse is all natural numbers, you are allowed to use P (a unary predicate symbol) and things like x +1.)

(III) 舉反例證無效性 Prove the invalidity of the following argument <u>by constructing a counterexample</u>. (10 points)

(6) 1. (∃y) Ay
2. (∃x) Bx /∴(∃z) (Az • Bz)

(Warning: Examples from daily life may be problematic. A mathematical one is preferable.)

(IV)形式證明 Prove the following arguments. (You may use the systems given below or some similar system. However, <u>semantic tableau system is less preferred and it may cause losing points (though *to write a tableau proof* is better than *to write nothing*.</u>)

(7) (x)(Qx • ~Px) /∴ (∃x) (~Px • Qx) (10 points)

(8) (∃x)(y) Rxy/∴(x)(∃y) Ryx (10 points)

(V)觀念題

(9) Explain the following concepts (giving a clear definition will suffice).

(9a) A set of sentences S is <u>satisfiable</u>. (5 points)

(9b) A set of sentences S is <u>consistent</u> (in first order logic). (5 points)

(10) Whether (9a) and (9b) are the same or different in first order logic? Explain. (10 points)

(VI) 舉例證明一致性　Show that the following three statements are not contradictory <u>by constructing an example</u>. (10 points)

(11)　1. (x) ~ (Rxx)

2. (x) (y) (z) [(Rxy $\wedge$ Ryz) $\rightarrow$ Rxz]

3. (x) ($\exists$y) Rxy

(Note: here R is a binary relation symbol.)

證明系統使用說明: 1. Most formal proof systems (Natural Deduction System, Gentzen's Sequent Calculus, the first system given below (or other similar systems), the second system given below (or other similar systems)) are acceptable, but <u>using ***semantic tableau system*** or other ***soundness-completeness unrecognizable proof systems*** may cause losing points</u> (*due to the undecidability of first order logic validity* (Church's theorem)).

2. <u>You are responsible for the **correct** way of using any of those systems or recalling meta-logical results (while not prohibited) in them</u>. That is, if you invalidly apply some inference rules (violating their constraints—though the constraints are not described below), it may cause losing points.

3. <u>It is OK to use different proof systems to solve different problems.</u> Within a problem, if one uses more than one system, it is subject to the grader's judgment.

You may use:

System I: This proof system contains implicational rules 1-8, equivalence rules 9-18, Conditional Proof (CP), Indirect Proof (IP), and 5 rules in predicate logic: one equivalence rule Quantifier Negation (QN), and 4 implicational rules: Universal Instantiation (UI), Existential Instantiation (EI), Universal Generalization (UG), Existential Generalization (EG). (*You are supposed to know the rules and their constraints if you use them*.)

1. MP $p \supset q$, $p / \therefore q$
2. MT $p \supset q$, $\sim q / \therefore \sim p$
3. DS $p \vee q$, $\sim p / \therefore q$ | $p \vee q$, $\sim q / \therefore p$
4. Simp $p \bullet q / \therefore p$; $p \bullet q / \therefore q$
5. Conj p, $q / \therefore p \bullet q$
6. HS $p \supset q$, $q \supset r / \therefore p \supset r$
7. Add $p / \therefore p \vee q$
8. CD $p \supset q$, $r \supset s$, $p \vee r / \therefore q \vee s$
9. DN $p :: \sim\sim p$
10. DeM $\sim(p \bullet q) :: \sim p \vee \sim q$; $\sim(p \vee q) :: \sim p \bullet \sim q$
11. Comm $(p \vee q) :: (q \vee p)$; $(p \bullet q) :: (q \bullet p)$
12. Assoc $[p \vee (q \vee r)] :: [(p \vee q) \vee r]$; $[p \bullet (q \bullet r)] :: [(p \bullet q) \bullet r]$
13. Dist $[p \bullet (q \vee r)] :: [(p \bullet q) \vee (p \bullet r)]$; $[p \vee (q \bullet r)] :: [(p \vee q) \bullet (p \vee r)]$
14. Contra $(p \supset q) :: (\sim q \supset \sim p)$
15. Impl $(p \supset q) :: \sim p \vee q$
16. Exp $[(p \bullet q) \supset r] :: [p \supset (q \supset r)]$
17. Taut $p :: (p \bullet p)$; $p :: (p \vee p)$
18. Equiv $(p \equiv q) :: [(p \supset q) \bullet (q \supset p)]$; $(p \equiv q) :: [(p \bullet q) \vee (\sim p \bullet \sim q)]$

(see next page for another system)

System II: (Due to the offering of graduate logic course in CCU…)

Sentential axioms:

(A1) $\alpha \rightarrow (\beta \rightarrow \alpha)$

(A2) $[\alpha \rightarrow (\beta \rightarrow \gamma)] \rightarrow [(\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)]$

(A3) $(\neg \beta \rightarrow \neg \alpha) \rightarrow (\alpha \rightarrow \beta)$

Quantifier axioms:

(B1) $\forall x\, (\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \forall x\, \beta)$

where x is not a free variable of α.

(B2) $(\forall x\ \alpha(x)) \rightarrow \alpha(t)$

where the term t is free (or substitutable) for x in $\alpha(x)$.

Equality axioms: x, y are variables, $t(\ldots)$ is any term, α is any atomic formula, i is any integer such that $1 \le i \le n$.

(C1) $x = x$

(C2) $x = y \rightarrow (t(v_1 \ldots v_{i-1} x v_{i+1} \ldots v_n) = t(v_1 \ldots v_{i-1} y v_{i+1} \ldots v_n))$

(C3) $x = y \rightarrow (\alpha(v_1 \ldots v_{i-1} x v_{i+1} \ldots v_n) \leftrightarrow \alpha(v_1 \ldots v_{i-1} y v_{i+1} \ldots v_n))$

Two inference rules

(MP) α and $\alpha \rightarrow \beta$ infers β.

(Gen) α infers $\forall x\, \alpha$.

(And for fairness, you are allowed to use Deduction Theorem while using this system. *You are supposed to know the rules and their constraints if you use them*.)