

臺灣綜合大學系統 109 學年度學士班轉學生聯合招生考試試題

科目名稱	邏輯	類組代碼	B20
		科目碼	B2092

※本項考試依簡章規定所有考科均「不可」使用計算機。

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In this test,

“¬” means “not”,

“∧” means “and”,

“∨” means “or”,

“→” means “if...then...”,

“↔” means “if and only if”,

“∀x” means “for all x” and

“∃x” means “for some x”.

### I. True or False

Please answer each question by writing **True** or **False**. No explanation is needed. (20 points; 2 points each)

- Two different predicates must have different interpretations (extensions).
- $P(a,b,a) \leftrightarrow P(b,a,b)$  logically implies  $a = b$ .
- If  $\exists x \exists y (P(x) \wedge P(y))$  is true in a model, then the domain of that model must contain at least two members.
- $\forall x (P(x) \rightarrow Q(x))$  is logically equivalent to  $\forall x P(x) \rightarrow \forall x Q(x)$ .
- $[(A \wedge B) \rightarrow C] \rightarrow [B \rightarrow (A \rightarrow (D \rightarrow C))]$  is a tautology.
- If every woman loves some man, then every woman loves some man who loves her. Hence no woman can love some man who does not love her.
- $P$  logically implies  $Q$  if and only if  $P$  logically implies  $P \rightarrow Q$ .
- Suppose  $A$  is not a contradiction. If  $A$  and  $B$  are inconsistent and  $A$  and  $C$  are inconsistent, then  $B$  and  $C$  must be inconsistent.
- Every premise in a valid argument must be true.
- Suppose  $P$  and  $S$  are consistent. Then  $P$  cannot logically imply  $Q$  if  $S$  and  $Q$  are inconsistent.

### II. Symbolization

Let “ $Lxy$ ” stand for “ $x$  loves  $y$ ”,

“ $Hxy$ ” stand for “ $x$  hates  $y$ ”,

“ $Px$ ” stand for “ $x$  is a philosopher”

and “ $Kx$ ” stand for “ $x$  is a king”.

Please symbolize the following two sentences. (20 points; 10 points each)

- No philosopher can be a king unless some king hates another king.
- There are exactly two philosophers who love each other.

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III. Please give counterexamples to the following three **invalid** arguments. (30 points; 10 points each)

- (1)  $\neg(A \leftrightarrow B) \leftrightarrow C, B \rightarrow (C \rightarrow D) \therefore (D \wedge A) \vee (B \leftrightarrow C)$
- (2) All good things will come to an end. No dictatorship is a good thing. Consequently, some dictatorships never come to an end.
- (3)  $\exists x(Px \rightarrow \forall yRy) \therefore \exists xPx \rightarrow \forall yRy$

IV. Please prove the following two **valid** arguments (all formal proof systems are acceptable; just make your proofs as clear as possible). (30 points; 15 points each)

- (1)  $A \vee \neg B, (B \rightarrow C) \rightarrow D \therefore (A \vee D)$
- (2)  $\therefore \exists y[R(x, y) \rightarrow \exists x \forall z R(x, z)]$